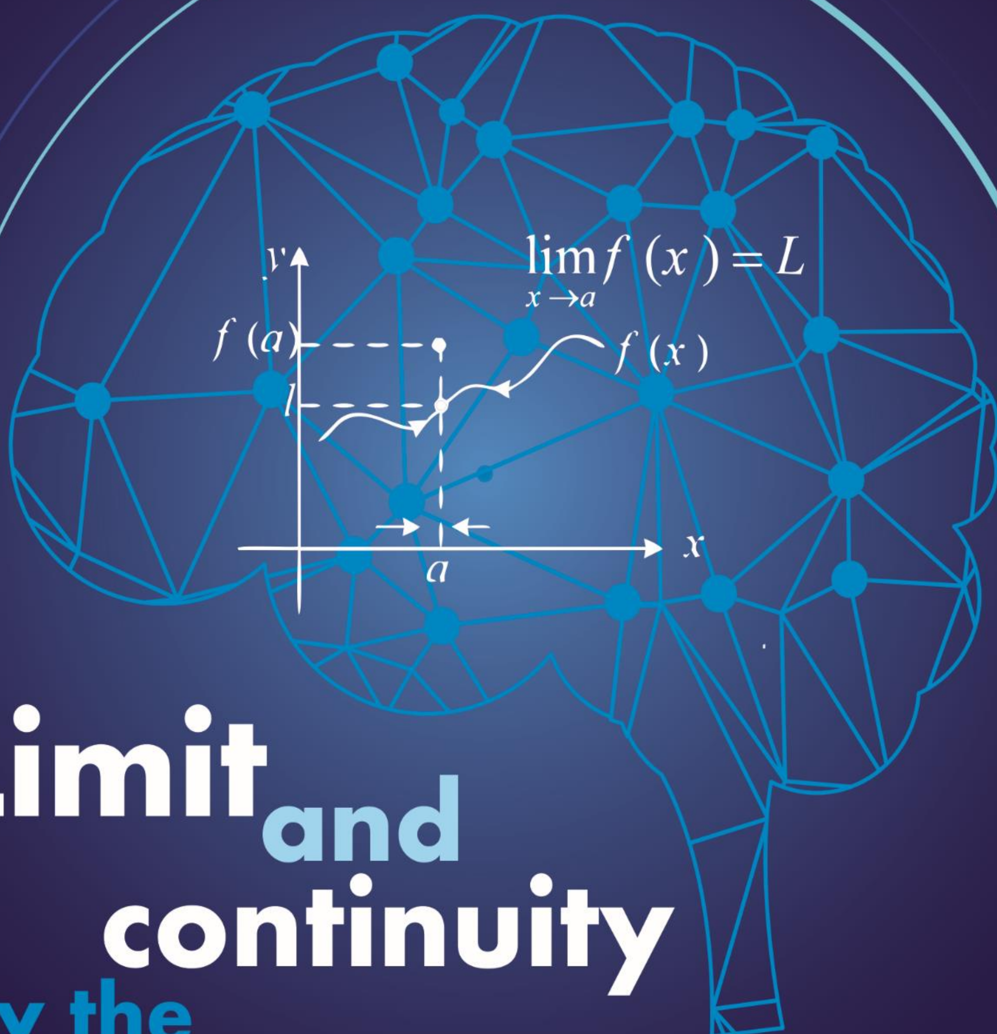

Limit and continuity by the mind table method

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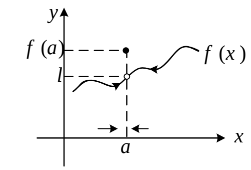
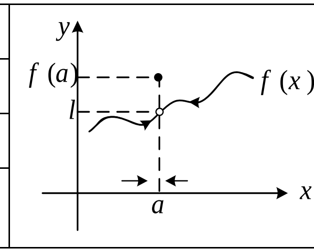
<table>
<tr><th colspan="8">Limit and Continuity Textbook
(Please note the questions related to the points after studying it)</th></tr>
<tr><th>Row</th><th>Title</th><th colspan="5">Description</th><th>Shape/diagram / formula</th></tr>
<tr><td rowspan="4">1</td><td rowspan="4">Definition of limit</td><td rowspan="2">1</td><td>Assumption</td><td colspan="3">Let $f(x)$ be a function defined on an interval that contains $x = a$, except possibly at $x = a$.</td><td rowspan="4">$\lim_{x \to a} f(x) = L$</td></tr>
<tr><td>Result</td><td colspan="3">Then we say that, $\lim_{x \to a} f(x) = L$</td></tr>
<tr><td rowspan="2">2</td><td>Assumption</td><td colspan="3">if for every number $\varepsilon > 0$</td></tr>
<tr><td>Result</td><td colspan="3">there is some number $\delta > 0$ such that $\lvert f(x) - L \rvert < \varepsilon$ whenever $0 < \lvert x - a \rvert < \delta$</td></tr>
<tr><td>2</td><td>Conditions for the existence of the limit</td><td colspan="5">1- f Defined in the neighborhood of a.
(Very close points that are in domain)
2- Left hand limit = right hand Limit</td><td></td></tr>
<tr><td rowspan="4">3</td><td rowspan="4">Calculate the limit</td><td colspan="2">first step</td><td colspan="4">Place a in f.</td></tr>
<tr><td colspan="2" rowspan="3">The result</td><td>number</td><td colspan="3">answer = number</td></tr>
<tr><td>divided by zero
or negative value under the radical</td><td colspan="3">Must be review</td></tr>
<tr><td>ambiguous situations</td><td colspan="3">fixing ambiguity</td></tr>
<tr><td>4</td><td>The concept of absolute answer</td><td colspan="6">As x moves closer to a, f approaches L.
In particular cases, f in neighborhood of a is exactly equal to L.
Consequently, it is called absolute or real L.</td></tr>
<tr><td>5</td><td>Functions that have absolute limit values</td><td colspan="6">Functions that have horizontal diagram in different intervals.
(have zero derivatives)
For example { Constant function
Ceiling function</td></tr>
<tr><td rowspan="2">6</td><td rowspan="2">Zero in denominator</td><td colspan="3">Absolute zero
(with no sign)</td><td colspan="3">There is no answer</td></tr>
<tr><td colspan="3">relative (partial) zero</td><td colspan="3">1- Determine sign of the numerator and denominator.
2- Divide the signs
3- { If the result was negative, the answer $= -\infty$
If the result was positive, the answer $= +\infty$</td></tr>
</table>

7	L^{+} or L^{-}	f in	a^{+}	strictly increasing function (derivative sign) $\Rightarrow f(a^{+}) = L^{+}$ strictly decreasing function (derivative sign) $\Rightarrow f(a^{+}) = L^{-}$	
			a^{-}	strictly increasing function (derivative sign) $\Rightarrow f(a^{-}) = L^{-}$ strictly decreasing function (derivative sign) $\Rightarrow f(a^{-}) = L^{+}$	
8	$\lim_{x \to a^{\pm}} \sqrt[n]{f(x)}$	1- { Order of a radical $\rightarrow$ even; radicand { Negative value / Or negative zero } $\Rightarrow$ no answer 2- Other cases, the answer is acceptable			
9	Calculation items of the left and the right limit	1- Ceiling functions; when the bracket approach to integer number. 2- Absolute value functions; when bracket approach zero. 3- The radical functions of even order; when radicand approach zero. 4- Fractional functions; when the denominator approach zero, but the numerator is not equal to zero. 5- Piecewise functions; when the limit is requested at the changed point of sub-function.			
10	Auxiliary points for calculating the limit	$0 < a < 1 \Rightarrow \log_a(0^{+}) = +\infty$ $a > 1 \Rightarrow \log_a(0^{+}) = -\infty$			
11		$a > 1 \Rightarrow a^{+\infty} = +\infty, a^{-\infty} = 0$ $0 < a < 1 \Rightarrow a^{+\infty} = 0, a^{-\infty} = +\infty$			
12		$\sqrt[2n]{\text{Every negative term including negative zero}}$ = no answer			
13		$\dfrac{\text{Any desired term including partial zero}}{\text{Absolute zero}}$ = no answer			
14		$\dfrac{\text{Absolute zero}}{\text{Any defined term except absolute zero}}$ = 0			
15		Partial or absolute zero $\times$ bounded term = 0			
16		Every term opposite zero $\times$ bounded term = no answer			
17		Absolute zero $\times \infty$ = 0			
18		$(\text{Absolute one})^{\infty} = 1$			
19		$(\infty)^{\text{Absolute zero}} = 1$			

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20		$(\text{Partial zero})^{\text{Absolute zero}} = 1$
21		$(\text{Absolute zero})^{\text{Positive partial zero}} = 0$
22		$(\text{Absolute zero})^{\text{Negetive partial zero}} = \text{no answer}$
23		$(\text{Positive partial zero})^{+\infty} = \text{no answer}$
24		$(\text{Negetive partial zero})^{+\infty} = \text{no answer}$

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Solving indeterminate forms using method of (McLaren)

Row	lim	$f(x)$	imbed	Denominator equivalence	Numerator equivalence	Equivalence again or simplification	Embed or simplification	The final answer
1	$x\to 0$	$\frac{\sin x - xe^{x^2}+\frac{7x^3}{6}}{\sin^2 x \sin x^3}$	$=\frac{\circ}{\circ}$	$\sin^2 x \sim x^2$ $\sin^3 x \sim x^3$	$\sin x=(x-\frac{x^3}{3!}+\frac{x^5}{5!}-...)$ $e^t=1+t+\frac{t^2}{2!}+....$	$\sim\frac{(x-\frac{x^3}{3!}+\frac{x^5}{5!})-x(1+x^2+\frac{(x^2)^2}{2!})+\frac{7x^3}{6}}{x^5}$	$=\frac{(-\frac{x^3}{3!}-x^3+\frac{x^5}{5!})-(\frac{x^5}{2!})+\frac{7x^3}{6}}{x^5}$ $=\frac{(-\frac{7x^3}{6}+\frac{7x^3}{6}+\frac{x^5}{5!}-\frac{x^5}{2!})}{x^5}$	$=\frac{-59}{120}$
2	$x\to 1^+$	$\frac{x-\cosh\sqrt{x-1}}{x-1}$	$=\frac{\circ}{\circ}$	x	$\cosh t=1+\frac{t^2}{2!}+\frac{t^3}{4!}+\cdots\Rightarrow$ $\cosh\sqrt{x-1}=1+\frac{1}{2}(x-1)+\cdots$	$\sim\frac{x-(1+\frac{1}{2}(x-1))}{x-1}$ $=\frac{x-1-\frac{1}{2}x+\frac{1}{2}}{x-1}=\frac{\frac{1}{2}x-\frac{1}{2}}{x-1}$	$=\frac{\frac{1}{2}(x-1)}{x-1}$	$=\frac{1}{2}$
3	$x\to 0$	$\frac{e^{ax}-e^x-2ax}{x^2}=0$	$=\frac{\circ}{\circ}$	x^2	$e^{ax}=1+ax+\frac{1}{2}(ax)^2$ $e^x=(1+x+\frac{x^2}{2})$	$\sim\frac{1+ax+\frac{1}{2}(ax)^2-1-x-\frac{x^2}{2}-2ax}{x^2}$	$=\frac{-(a+1)x+\frac{1}{2}(a^2-1)x^2}{x^2}=0$	$a+1=0,$ $a^2-1=0$ $\Rightarrow a=-1$
4	$x\to 0$	$\frac{x+\ln(1-x)}{\sin^2 x}$	$=\frac{\circ}{\circ}$	$\sin^2 x\sim x^2$	$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-...$	$\sim\frac{x+(-x-\frac{x^2}{2})}{x^2}$	-	$=-\frac{1}{2}$
5	$x\to 0^-$	$\frac{2^{\|x\|}+x\ln 2-1}{\sin 2x}$	$=\frac{\circ}{\circ}$	$\sin 2x\sim 2x$	$\|x\|=-x$ $a^x-1\sim x\ln a$	$\sim\frac{-x\ln 2+x\ln 2}{2x}$	-	$=0$
6	$x\to 0$	$\frac{\sin x-x\cos x-bx^3}{x^4}$	$=\frac{\circ}{\circ}$	x^4	$\sin x\sim(x-\frac{x^3}{6}+\frac{x^5}{120})$ $\cos x\sim(1-\frac{x^2}{2}+\frac{x^4}{24})$	$\sim\frac{(x-\frac{x^3}{6}+\frac{x^5}{120})-x(1-\frac{x^2}{2}+\frac{x^4}{24})-bx^3}{x^4}$ $=\frac{\frac{x^3}{3}-bx^3}{x^4}+\frac{\frac{4x^5}{120}}{x^4}=\frac{\frac{x^3}{3}-bx^3}{x^4}+\frac{x}{30}$	$\frac{(\frac{1}{3}-b)x^3}{x^4}+0$ $\Rightarrow\frac{1}{3}-b=0$	$b=\frac{1}{3}$

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Answer of one to the power of infinity questions

Row	lim	$f(x)^{g(x)}$	Embed	$(1^{\infty} \sim e^{g(f-1)})$	Simplify or equivalence again	Simplify or equivalence three times	The final answer
1	$x \to 0$	$(\cos 2x)^{\cot^2 x}$	$=1^{\infty}$	$=e^{\cot^2 x(\cos 2x-1)}$	$=e^{\frac{\cos 2x-1}{\tan^2 x}}$	$=e^{\frac{1-\frac{(-2x)^2}{2}-1}{x^2}}$	$=e^{-2}$
2	$x \to 0^+$	$(1+\sin 5x)^{\cot x}$	$=1^{\infty}$	$=e^{\cot x(1+\sin 5x-1)}$	$=e^{\frac{\sin 5x}{\tan x}}$	$=e^{\frac{5x}{x}}$	$=e^{5}$
3	$x \to 0$	$(\cos\sqrt{x})^{\frac{1}{x}}$	$=1^{\infty}$	$=\mathrm{e}^{\frac{1}{x}(\cos\sqrt{x}-1)}$	$=e^{\frac{1-\frac{x}{2}-1}{x}}$	$=e^{-\frac{1}{2}}$	$=\frac{1}{\sqrt{e}}$
4	$x \to 0$	$(1+2x)^{\frac{3}{\sin 2x}}$	$=1^{\infty}$	$=e^{\frac{3}{\sin 2x}(1+2x-1)}$	$=e^{\frac{6x}{\sin 2x}}$	$=e^{\frac{6x}{2x}}$	$=e^{3}$
5	$x \to 0$	$(\frac{\sin x}{x})^{\frac{1}{x^2}}$	$=1^{\infty}$	$=e^{\frac{1}{x^2}(\frac{\sin x}{x}-1)}$	$=e^{(\frac{\sin x-x}{x^3})}$	$=e^{(\frac{(x-\frac{x^3}{6})-x}{x^3})}$	$=e^{-\frac{1}{6}}$
6	$x \to \infty$	$(\frac{2x+3}{2x+8})^{x}$	$=1^{\infty}$	$=e^{x(\frac{2x+3}{2x+8}-1)}$	$=e^{\frac{-5x}{2x+8}}$	$=e^{\frac{-5x}{2x}}$	$=e^{-\frac{5}{2}}$
7	$x \to \infty$	$(\frac{2x-3}{2x+5})^{2x+1}$	$=1^{\infty}$	$=e^{(2x+1)(\frac{2x-3}{2x+5}-1)}$	$=e^{\frac{-8(2x+1)}{2x+5}}$	$=e^{\frac{-16x}{2x}}$	$=e^{-8}$
8	$x \to \frac{\pi}{4}$	$(\tan x)^{\tan 2x}$	$=1^{\infty}$	$=\mathrm{e}^{\tan 2x(\tan x-1)}$	$=\mathrm{e}^{\frac{2\tan x}{1-\tan^2 x}(\tan x-1)}$ $=\mathrm{e}^{\frac{2\tan x(\tan x-1)}{(1+\tan x)(1-\tan x)}}$	$=\mathrm{e}^{\frac{-2\tan x}{(1+\tan x)}}$ $=\mathrm{e}^{-\frac{2\tan\frac{\pi}{4}}{(1+\tan\frac{\pi}{4})}}=e^{-1}$	$=\frac{1}{e}$
9	$x \to +\infty$	$(\frac{1+x}{5+x})^{3x+2}$	$=1^{\infty}$	$=e^{(3x+2)(\frac{1+x}{5+x}-1)}$	$=e^{\frac{-4(3x+2)}{5+x}}$	$=e^{-\frac{12x}{x}}$	$=e^{-12}$
10	$n \to +\infty$	$(\frac{2n^2+3n}{2n^2-n+1})^{n}$	$=1^{\infty}$	$=e^{n(\frac{2n^2+3n}{2n^2-n+1}-1)}$	$=e^{\frac{n(4n-1)}{2n^2-n+1}}$	$=e^{\frac{4n^2}{2n^2}}$	$=e^{2}$
11	$x \to 0$	$(x-\sin x+\cos 2x)^{x^{-2}}$	$=1^{\infty}$	$=e^{x^{-2}(x-\sin x+\cos 2x-1)}$ $=e^{\frac{x-\sin x+\cos 2x-1}{x^2}}$	$=e^{\frac{x-(x-\frac{x^3}{6})+(1-\frac{4x^2}{2})-1}{x^2}}$	$=e^{\frac{x-x+\frac{x^3}{6}+1-\frac{4x^2}{2}-1}{x^2}}=e^{\frac{-\frac{4x^2}{2}}{x^2}}=e^{-2}$	$=\frac{1}{\sqrt{e}}$

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Row	$f(x)=[g(x)]$	interval	$g(x)$ interval	Discontinuity candidate (such value of x where $g(x)$ is integer)	Local minimum	The beginning of interval or any objects in the interval. Check the right-continuity	The ending of interval or any objects in the interval. Check the left-continuity	Discontinuity points
1	$f(x)=[(x-1)^2]$	$(0,2)$	$0<x<2$ $-1<x-1<1$ $0\le(x-1)^2<1$	$(x-1)^2\in\mathbb{Z}$ $\Rightarrow(x-1)^2=0$ $\Rightarrow x=1$	$x=1$	$\lim_{x\to0^+} f(x)=f(0)$ right-continuous	$\lim_{x\to2^-} f(x)=f(2)$ left-continuous	Doesn't have
2	$f(x)=[x]+[2x]$	$[0,2]$	$0\le x\le2$ $0\le 2x\le4$	$2x\in\mathbb{Z}$ $\Rightarrow\ 2x=0,1,2,3,4$ $\Rightarrow\ x=\frac{1}{2},1,\frac{3}{2},2$	Doesn't have	$\lim_{x\to0^+} f(x)$ $=0+[0^+]=0\ne f(0)$ right-continuous	$\lim_{x\to2^-} f(x)$ $=2+[4^-]=5\ne f(2)=6$ left-continuous	$x=0,\frac{1}{2},1,\frac{3}{2}.2$
3	$f(x)=x^2+[x^2]$	$[0,2]$	$0\le x\le2$ $0\le x^2\le4$	$x^2\in\mathbb{Z}$ $\Rightarrow\ x^2=1,2,3$ $\Rightarrow\ x=1,\sqrt{2},\sqrt{3}$	Doesn't have	$\lim_{x\to0^+} f(x)$ $=0+[0^+]=0=f(0)$ right-continuous	$\lim_{x\to2^-} f(x)$ $=4+[4^-]=7\ne f(2)=8$ left-continuous	$x=1,\sqrt{1},\sqrt{2},2$
4	$f(x)=[4\sin^2 x]$	$[-2,2]$	$-1\le\sin x\le1$ $0\le\sin^2 x\le1$ $0\le4\sin^2 x\le4$	$4\sin^2 x\in\mathbb{Z}$ $\Rightarrow 4\sin^2 x=0,1,2,3,4$ $\Rightarrow x=0,\pm\frac{\pi}{6},\pm\frac{\pi}{4},\pm\frac{\pi}{3},\pm\frac{\pi}{2}$	$x=0$	$f(-2)\notin\mathbb{Z}$	$f(2)\notin\mathbb{Z}$	$x=\pm\frac{\pi}{6},\pm\frac{\pi}{4},\pm\frac{\pi}{3},\pm\frac{\pi}{2}$
5	$f(x)=x-[x]$ $g(x)=\sin\pi x$ $gof=\sin\pi(x-[x])$	$[-2,2]$	$x=-1,0,1$	$x=-2,-1,0,1,2$	Doesn't have	$x\to-2^+:\sin(-2\pi)=0=gof(-2\pi)$ $x\to-1^+:\sin(-\pi)=0=gof(-1)$ $x\to0^+:\sin(0)=0=gof(0)$ $x\to1^+:\sin\pi=0=gof(\pi)$	$x\to-1^-:\sin(-\pi)=0=gof(-1)$ $x\to0^-:\sin(0)=0=gof(0)$ $x\to1^-:\sin\pi=0=gof(\pi)$ $x\to2^-:\sin(2\pi)=0=gof(2\pi)$	Doesn't have
6	$f(x)=x-[x]$	$\|x\|\le2$ $[-2,2]$	$x=-2,-1,0,1,2$	$x=-2,-1,0,1,2$	Doesn't have	$\lim_{x\to-2^+} f(x)$ $=-2-[-2^+]$ $=-1=f(-2)$	$\lim_{x\to2^-} f(x)$ $=4-[4^-]=1\ne f(2)=0$	$x=-1,0,1,2$

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